

## CKM PARAMETERS AND RARE B DECAYS

F. FORTI

*INFN-Pisa, L.go Pontecorvo, 3, 56127 Pisa, Italy*  
*E-mail: Francesco.Forti@pi.infn.it*

Measurements of the angles and sides of the unitarity triangle and of the rates of rare  $B$  meson decays are crucial for the precise determination of Standard Model parameters and are sensitive to the presence of new physics particles in the loop diagrams. In this paper the recent measurements performed in this area by *BABAR* and *Belle* will be presented. The direct measurement of the angle  $\alpha$  is for the first time as precise as the indirect determination. The precision of the  $|V_{ub}|$  determination has improved significantly with respect to previous measurement. New limits on  $B \rightarrow \tau\nu$  decays are presented, as well as updated measurements on  $b \rightarrow s$  radiative transitions and a new observation of  $b \rightarrow d\gamma$  transition made by *Belle*.

## 1 Introduction

In the Standard Model (SM), the interaction between up-type and down-type quarks is described by a unitarity matrix called the Cabibbo-Kobayashi-Maskawa matrix (in brief CKM).<sup>1,2</sup> This matrix can be parametrized with 3 real angles and one complex phase, which gives rise to CP violation. A widely used parametrization of the matrix<sup>3,4</sup> uses the four parameters  $A, \lambda, \bar{\rho}, \bar{\eta}$ , with  $\bar{\eta}$  controlling the CP violation in this framework. The unitarity of the CKM matrix imposes 9 complex relations amongst the matrix elements, one of which is given by

$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0,$$

where  $V_{qq'}$  is the matrix element relating the quark  $q$  and  $q'$ . This relation can be represented as a triangle (called the unitarity triangle) in the complex  $\bar{\rho}, \bar{\eta}$  plane, as shown in Fig. 1.  $B$ -meson decays are sensitive probes to measure both the angles and sides of the unitarity triangle and can unveil physics beyond the SM. In fact, most  $B$  decay amplitudes receive contributions from diagrams containing loops, where the presence of new particles can be detected through effects on the branching ratios, asymmetries, or spectra. Another possible route to detecting new physics is the high precision measurement of the unitarity triangle parameters to uncover

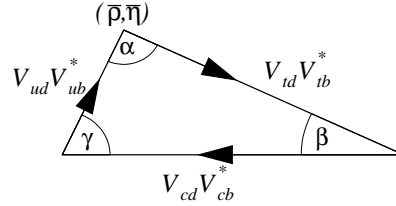


Figure 1. Unitarity triangle.

any inconsistency among them or between different determinations of the same parameter.

After having clearly established CP-violation in the  $B$  sector, the *BABAR* and *Belle* experiments are now pursuing an extended program of precision measurements of the unitarity triangle parameters and of rare  $B$  decays, taking advantage of the very large data sample collected at the B-Factories. The recent results of this measurement program are reported at this conference in two papers. The measurement of  $\sin 2\beta$  and the direct CP violation measurements are presented by Kazuo Abe. In this paper, after introducing the *BABAR* and *Belle* experiments in Sec. 2, I will cover the  $\alpha$  measurements in Sec. 3 and the  $|V_{ub}|$  and  $|V_{cb}|$  measurements in Sec. 4. The rest of the paper will be devoted to rare decays:  $B \rightarrow \tau\nu$  in Sec. 5,  $b \rightarrow s\gamma$  in Sec. 6, and  $b \rightarrow d\gamma$  in Sec. 7. I will finally give some concluding remarks in Sec. 8.

## 2 The B-Factory experiments and datasets

The data used in the analyses presented in this paper have been collected with the *BABAR* detector at the PEP-II machine at SLAC and with the Belle detector at the KEKB machine in the KEK laboratory between 1999 and 2005. Both detectors, whose detailed description can be found elsewhere,<sup>5,6</sup> have been designed and optimized to study time-dependent CP-violation in  $B$  decays at the  $\Upsilon(4S)$  resonance. Their major components are: a vertexing and tracking system based on silicon and gas detectors; a particle identification system; an electromagnetic calorimeter based on CsI(Tl) crystals operating within a 1.5 T magnetic field; an iron flux return located outside of the coil, instrumented to detect  $K_L^0$  and identify muons. The  $\Upsilon(4S)$  resonance decays most of the time in a pair of  $B$ -mesons, either  $B^+B^-$  or  $B^0\bar{B}^0$ , which acquire a boost thanks to the asymmetry of the beam energies: 9 GeV  $e^-$  on 3.1 GeV  $e^+$  for PEP-II and 8 GeV  $e^-$  on 3.5 GeV  $e^+$  for KEKB. Because of this boost, the decay vertices of the two mesons are separated, thus allowing their individual determination and the measurement of time-dependent CP asymmetries. In these analyses, the signal  $B$  is reconstructed in a CP-eigenstate (such as  $B \rightarrow \pi\pi$ ) while the other  $B$  (the tagging  $B$ ) is reconstructed in a decay mode that allows the determination of its flavor at the time of decay, such as exclusive hadronic or semileptonic modes, or inclusive modes with a lepton or a kaon, whose sign carries the information of the  $B$  flavor.

The data samples used in the measurements presented in this paper vary for the two experiments. Most measurements are based on  $232 \times 10^6 B\bar{B}$  pairs for *BABAR* and  $275 \times 10^6 B\bar{B}$  for Belle, but there are several results obtained with smaller statistics, while Belle performed the  $b \rightarrow d\gamma$  analysis with  $385 \times 10^6 B\bar{B}$ .

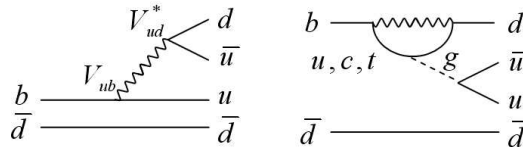


Figure 2. The tree (left) and penguin (right) diagrams contributing to charmless  $B$  decays  $B^0 \rightarrow \pi^+\pi^-$ ,  $B^0 \rightarrow \rho^+\pi^-$  and  $B^0 \rightarrow \rho\rho$ .

## 3 Determination of the angle $\alpha(\Phi_2)$

The angle  $\alpha$  is the relative phase of the  $V_{ub}$  and  $V_{td}$  CKM matrix elements and can be measured in the charmless  $B$  decays  $B \rightarrow \pi\pi$ ,  $B \rightarrow \rho\pi$  and  $B \rightarrow \rho\rho$  which arise from tree-level  $b \rightarrow u(\bar{u}d)$  transitions (Fig. 2, left). A complication to this approach is the presence of loop level penguin diagrams leading to the same final states (Fig. 2, right), which introduce different CKM matrix elements. While in the absence of penguin contribution, the measurement of time dependent CP asymmetries in neutral  $B$  charmless decays would directly yield the angle  $\alpha$ , the interference between tree and penguin diagrams obscures the simple relationship between CP observables and the angle  $\alpha$  and requires the development of specific techniques to disentangle the penguin contribution.

Time-dependent CP asymmetries arise from the interference of two possible paths reaching the same final state:  $B \rightarrow f$  and  $B \rightarrow \bar{B} \rightarrow f$ , and can be expressed in terms of the complex parameter  $\lambda_f = \eta_f \frac{p}{q} \frac{\bar{A}}{A}$ , where  $A = |\langle f|T|B^0\rangle|$ ,  $\bar{A} = |\langle f|T|\bar{B}^0\rangle|$ ,  $\eta_f$  is the CP eigenvalue of the final state and  $q, p$  are the parameters describing how  $B^0$  and  $\bar{B}^0$  mix to form the mass eigenstates. The time dependent CP asymmetry follows

$$A_{CP}(\Delta t) = S_f \sin(\Delta m \Delta t) + C_f \cos(\Delta m \Delta t),$$

where  $S_f = 2 \frac{\Im(\lambda_f)}{1+|\lambda_f|^2}$  measures the CP violation arising from the interference of the decays with and without mixing, and  $C_f = \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}$  measures the direct CP violation

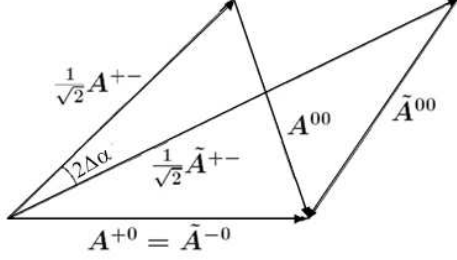


Figure 3. Isospin triangles for the charmless  $B$  decays  $B^0 \rightarrow \pi^+\pi^-$ ,  $B^0 \rightarrow \rho\rho$ .

in the decay. For the tree diagram in (Fig. 2, left)

$$\lambda_f = \eta_f \frac{V_{tb}^* V_{td} V_{ub} V_{ud}^*}{V_{tb} V_{td}^* V_{ub}^* V_{ud}} = \eta_f e^{2i\alpha},$$

with  $C_f = 0$  and  $S_f = \sin(2\alpha)$ . In the presence of the penguin diagram, the expression becomes:

$$\lambda_f = \eta_f e^{2i\alpha} \frac{T + P e^{+i\gamma} e^{i\delta}}{T + P e^{-i\gamma} e^{i\delta}}$$

where  $T$  and  $P$  are the tree and penguin amplitudes, and  $\delta$  is the strong phase. The effect of penguin diagram interference is the possibility of direct CP violation ( $C_f \propto \sin \delta$ ) and a shift  $\Delta\alpha$  in the measurement of the angle  $\alpha$ :  $S_f = \sqrt{1 - C_f^2} \sin(2\alpha_{\text{eff}})$  with  $\Delta\alpha = \alpha_{\text{eff}} - \alpha$ .

Isospin relations amongst rates of the various  $B \rightarrow \pi\pi$  and  $B \rightarrow \rho\rho$  decays can be used<sup>7</sup> to extract the shift  $\Delta\alpha$ . The isospin analysis involves the separate measurement of  $B^0$  and  $\bar{B}^0$  decay rates into  $h^+h^-$  ( $h$  indicates either  $\pi$  or  $\rho$ ) and  $h^0h^0$ , as well as the measurement of the rate of the charged  $B$  decay  $B^{+(-)} \rightarrow h^{+(-)}h^0$ . Constructing a  $B^0$  and a  $\bar{B}^0$  triangle from the 6 amplitudes (Fig. 3) one can extract  $\Delta\alpha$  from the mismatch of the two triangles.

It has also been shown<sup>8</sup> that, in alternative to full isospin analysis, one can use the branching fractions for  $B \rightarrow h^0h^0$  and  $B \rightarrow h^+h^0$  averaged over meson and anti-meson to impose an upper bound on  $\Delta\alpha$ :

$$\sin^2 \Delta\alpha < \frac{\overline{\mathcal{B}}(B^0 \rightarrow h^0h^0)}{\overline{\mathcal{B}}(B^\pm \rightarrow h^\pm h^0)}$$

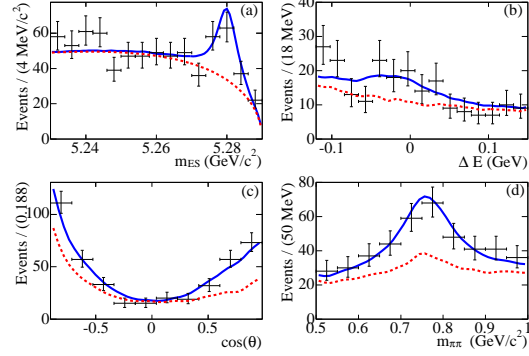


Figure 4. The distributions for the highest purity tagged events in the BABAR  $B \rightarrow \rho\rho$  analysis for the variables  $m_{ES}$  (a),  $\Delta E$  (b), cosine of the  $\rho$  helicity angle (c), and  $m_{\pi^\pm\pi^0}$  (d). The dotted lines are the sum of backgrounds and the solid lines are the full PDF.

Other relations have also been developed,<sup>9,10</sup> but with the current level of accuracy of the measurements none improves significantly over the above limit. The constraints on  $\alpha$  derived from a full isospin analysis in the  $\pi\pi$  channel<sup>11,12</sup> are very weak, as shown in Fig. 5 explained later in the text, mainly due to the fact that the branching ratio  $\mathcal{B}(B^0 \rightarrow \pi^0\pi^0) = (1.45 \pm 0.29) \times 10^{-6}$  (averaged by HFAG<sup>13</sup> on the basis of the BABAR<sup>14</sup> and Belle<sup>15</sup> measurements) is too large to be effective in setting the above limit, but is also too small for the full isospin analysis.

The  $\rho\rho$  channel has three polarization amplitudes, which introduce dilution in the measurement because they have different CP eigenvalues, and has been considered in the past as less promising than  $\pi\pi$ . Both BABAR and Belle have recently performed full analyses of this decay.<sup>16,17</sup> The charged  $\rho$  is reconstructed through the decay  $\rho^\pm \rightarrow \pi^\pm\pi^0$ , and the events are selected through a kinematical signal identification based on the beam-energy substituted mass (also known as beam constrained mass)  $m_{bc} \equiv m_{ES} = \sqrt{E_{\text{beam}}^{*2} - p_B^{*2}}$  and the energy difference between the reconstructed  $B$  and the beam

$\Delta E = E_B^* - E_{\text{beam}}^*$ . All quantities are computed in the CM frame. The distribution of these variables for the signal and the background is shown in Fig. 4

It is found that the fraction of longitudinal polarization ( $f_L$ ) in the  $\rho\rho$  final state is almost 100%, and that therefore there is no dilution effect in the measurement of  $\alpha$ . In addition, the  $\rho^+\rho^-$  and  $\rho^+\rho^0$  branching fractions are a factor of 5 larger than the corresponding ones in the  $\pi\pi$  decays, but at the same time the  $\rho^0\rho^0$  is not yet observed, with a relatively small limit on  $\Delta\alpha$ . The results are summarized in Table 1

Using the *BABAR* limit on  $\mathcal{B}(B^0 \rightarrow \rho^0\rho^0)$  and the average between the two experiments for the other quantities one arrives at a relatively stringent limit on  $\Delta\alpha$  ( $\Delta\alpha < 11^\circ$ ) and at the determination  $\alpha[\rho\rho] = (96 \pm 13)^\circ$ .

The isospin analysis has an intrinsic two-fold ambiguity that can be removed with a full time-dependent Dalitz plot analysis of the  $B \rightarrow \rho\pi$  decay.<sup>18</sup> Results on this analysis have been presented the ICHEP04 conference.<sup>19,20</sup>

The results of the three analysis are summarized in Fig. 5, where a combined fit<sup>21</sup> is also shown. The result of this combined fit is  $\alpha = (99_{-9}^{+12})^\circ$ . The result from the indirect measurement of  $\alpha$  obtained by fitting all the other CKM triangle measurements,  $\alpha[\text{CKM}] = (96_{-12}^{+11})^\circ$  is shown for comparison on the same plot. This is the first time that the direct measurement of  $\alpha$  has a better precision than the its indirect determination from the CKM triangle fit.

#### 4 Measurement of $|V_{ub}|$ and $|V_{cb}|$

The magnitude of the CKM matrix elements  $V_{ub}$  and  $V_{cb}$  can be extracted from the semileptonic decay rate of  $B$  mesons. At the parton level the decay rates for  $b \rightarrow u\ell\nu$  and  $b \rightarrow c\ell\nu$  can be calculated accurately; they are proportional to  $|V_{ub}|^2$  and  $|V_{cb}|^2$ , respectively, and depend on the quark masses,

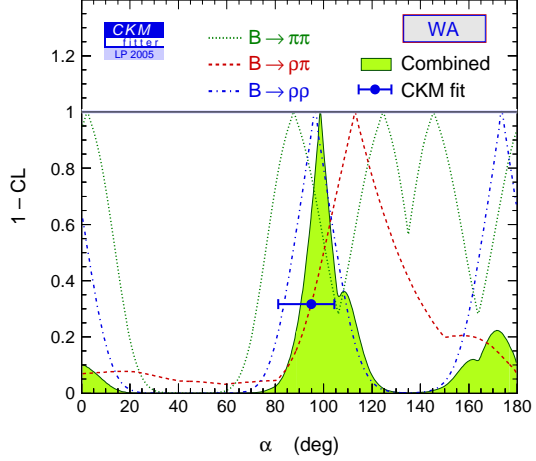


Figure 5. Alpha determination from the charmless  $B$  decays  $B^0 \rightarrow \pi^+\pi^-$ ,  $B^0 \rightarrow \rho\rho$  and  $B^0 \rightarrow \rho^+\pi^-$ . The dotted lines represent the results of the three individual analyses. The green area is the result of the combined fit. The CKM triangle fit independent determination of alpha, which is not included in the fit, is shown by the blue point.

$m_b$ ,  $m_u$ , and  $m_c$ . To relate measurements of the semileptonic decay rate to  $|V_{ub}|$  and  $|V_{cb}|$ , the parton-level calculations have to be corrected for effects of strong interactions, thus introducing significant theoretical uncertainties for both exclusive and inclusive analyses.

For of exclusive decays, the effect is parametrized by form factors(FF), such as in the simple case of the  $B \rightarrow \pi\ell\nu$  decay, neglecting the  $\pi$  mass:

$$\frac{d\Gamma(B^0 \rightarrow \pi^- \ell^+ \nu)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} |f_+(q^2)|^2 p_\pi^3,$$

where  $G_F$  is the Fermi constant,  $q^2$  is the invariant-mass squared of the lepton-neutrino system and  $p_\pi$  is the pion momentum in the  $B$  frame. The FF  $f_+(q^2)$  can be calculated with a variety of approaches based on quark model,<sup>22</sup> Light Cone Sum Rules,<sup>23</sup> and lattice QCD.<sup>24,25</sup> In inclusive decays, the main difficulty is to relate the partial rate obtained by the experimental event selection process to the matrix elements. This is a particularly serious issue for  $|V_{ub}|$ , where only a

Table 1. Summary of measurements for the  $B \rightarrow \rho\rho$  decays

Quantity	BABAR	Belle
$f_L$	$0.978 \pm 0.014^{+0.021}_{-0.029}$	$0.951^{+0.033+0.029}_{-0.039-0.031}$
$S_{\rho\rho,L}$	$-0.33 \pm 0.24^{+0.08}_{-0.14}$	$0.09 \pm 0.42 \pm 0.08$
$C_{\rho\rho,L}$	$-0.03 \pm 0.18 \pm 0.09$	$0.00 \pm 0.30^{+0.09}_{-0.10}$
$\mathcal{B}(B^0 \rightarrow \rho^+\rho^-) [10^{-6}]$	$30 \pm 4 \pm 5$	$24.4 \pm 2.2^{+3.8}_{-4.1}$
$\mathcal{B}(B^\pm \rightarrow \rho^+\rho^0) [10^{-6}]$	$22.5^{+5.7}_{-5.4} \pm 5.8$	$31.7 \pm 7.1^{+3.8}_{-6.7}$
$\mathcal{B}(B^0 \rightarrow \rho^0\rho^0) [10^{-6}]$	$< 1.1$	-

small fraction of the total rate can be determined experimentally because of the severe background rejection cuts. Heavy-Quark Expansions (HQEs)<sup>26</sup> have become a useful tool for calculating perturbative and non-perturbative QCD corrections and for estimating their uncertainties. These expansions contain parameters such as the  $b$  quark mass and the average Fermi momentum of the  $b$  quark inside the  $B$  meson. These parameters must be determined experimentally, for instance from the photon energy spectrum in  $B \rightarrow X_s\gamma$  decays and the spectrum of the hadronic mass in  $B \rightarrow X_c\ell\nu$  decays.

For the determination of  $|V_{cb}|$ , a global analysis of inclusive  $B$  decays has been performed,<sup>27</sup> leading to a very precise measurement:

$$|V_{cb}|_{\text{incl.}} = (41.4 \pm 0.6_{\text{exp}} \pm 0.1_{\text{th}}) \times 10^{-3}.$$

The measurement obtained from the world average of  $\mathcal{B}(B \rightarrow D^*\ell\nu)$ ,<sup>28</sup>

$$|V_{cb}|_{D^*\ell\nu} = (41.3 \pm 1.0_{\text{exp}} \pm 1.8_{\text{th}}) \times 10^{-3},$$

is fully compatible, although less precise.

These accurate measurements demonstrate the rapid experimental and theoretical advancements in these area.

#### 4.1 $b \rightarrow u\ell\nu$ inclusive decays.

Several methods have been used to isolate inclusive  $b \rightarrow u\ell\nu$  decays from the much more frequent  $b \rightarrow c\ell\nu$  decays.

In the lepton endpoint method<sup>29,30</sup> one uses the fact that, due to the mass difference between  $c$  and  $u$  quarks, the lepton spectrum in the  $b \rightarrow u$  transition extends to slightly higher energies than in the  $b \rightarrow c$  decays. The lepton momentum window is typically  $1.9 < p_{\text{lept}} < 2.6 \text{ GeV}/c$ , and a selection is applied on the basis of event shape variables and missing momentum. The background remains in any case significant with typically  $S/B \approx 1/14$ .

One can refine the selection by using a  $q^2$ -dependent electron energy cut (the  $E_e - q^2$  method)<sup>31</sup> where the neutrino momentum is estimated from the event missing momentum and  $q^2$  is calculated from  $q^2 = (p_e + p_\nu)^2$ . For each  $E_e$  and  $q^2$  one can calculate the maximum kinematically allowed hadronic mass square  $s_h^{\text{max}}$  and veto  $b \rightarrow c\ell\nu$  decays by requiring  $s_h^{\text{max}} < 3.5 \text{ GeV}^2 \approx m_D^2$ . This technique significantly improves the S/B ratio to about 1/2. Figure 6 shows the electron energy and  $s_h^{\text{max}}$  spectra, along with signal and sideband regions.

Reconstructing the other  $B$  in the event in an exclusive channel allows the direct reconstruction of the hadronic system (called

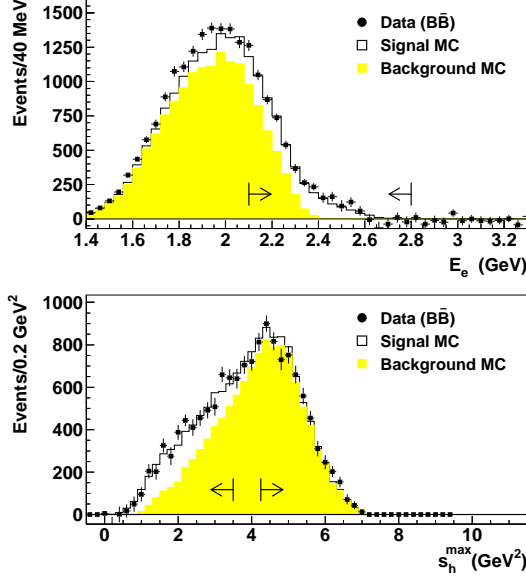


Figure 6. The electron energy,  $E_e$ , and  $s_h^{\max}$  spectra in the  $\Upsilon(4S)$  frame for continuum-subtracted data and simulated  $B\bar{B}$  events satisfying all the selection criteria except for the variable shown. The arrows denote the signal and sideband regions.

$X$ ) produced in  $b \rightarrow u\ell\nu$  decays by assigning all the remaining particles to it. BABAR uses the mass of the hadronic system to perform a 2-dimensional fit for the partial branching fraction in the area  $\{M_X < 1.7 \text{ GeV}/c^2, q^2 > 8 \text{ GeV}^2\}$ ,<sup>32</sup> while Belle also introduces the variable  $P_+ \equiv E_X - |\mathbf{p}_X|$ , where  $E_X$  and  $\mathbf{p}_X$  are the energy and 3-momentum of the hadronic system, analyzing data in three kinematical regions  $M_X < 1.7 \text{ GeV}/c^2$ ,  $\{M_X < 1.7 \text{ GeV}/c^2, q^2 > 8 \text{ GeV}^2\}$ , and  $P_+ < 0.66 \text{ GeV}/c$ .<sup>33</sup>

The extraction of  $|V_{ub}|$  from these partial branching fractions involves the determination of HQE parameters, which can be done following a variety of schemes and using different physical processes.<sup>34</sup> This extraction is the object of a very active discussion with the goal of improving the precision of the measurement. A summary of  $|V_{ub}|$  inclusive determinations based on HQE parameters de-

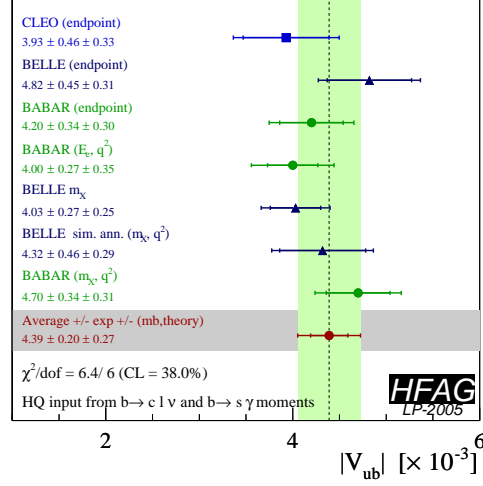


Figure 7. Summary and average of inclusive  $|V_{ub}|$  determinations using HQE parameters extracted from  $B \rightarrow X_s\gamma$  and  $B \rightarrow X_c\ell\nu$  moments.

rived from the moments of the photon energy spectrum in  $B \rightarrow X_s\gamma$  decays and from the hadronic-mass and lepton-energy moments in  $B \rightarrow X_c\ell\nu$  decays is shown<sup>13</sup> in Fig. 7:

$$|V_{ub}|_{\text{incl.}} = (4.39 \pm 0.20_{\text{exp}} \pm 0.27_{\text{th}}) \times 10^{-3}.$$

An alternative determination, using HQE parameters<sup>35</sup> obtained fitting the Belle  $B \rightarrow X_s\gamma$  photon energy spectrum, yields:<sup>29</sup>

$$|V_{ub}|_{\text{incl.}} = (5.08 \pm 0.47_{\text{exp}} \pm 0.48_{\text{th}}) \times 10^{-3}.$$

#### 4.2 $B \rightarrow \pi\ell\nu, \rho\ell\nu$ decays.

Various methods have been devised to isolate exclusive  $B \rightarrow \pi\ell\nu, \rho\ell\nu$  decays from the large backgrounds from  $b \rightarrow c\ell\nu$  and continuum events. Estimating the neutrino momentum from the missing momentum in the event allows the usage of the mass of the  $B$  candidate  $m_{ES}$  as a discriminating variable. In addition, one can analyze the data in bins of  $q^2$  (three bins for CLEO<sup>36</sup> and five bins for BABAR<sup>37</sup>) and measure the  $q^2$  dependance of the form factor, thus discriminating among theoretical models.

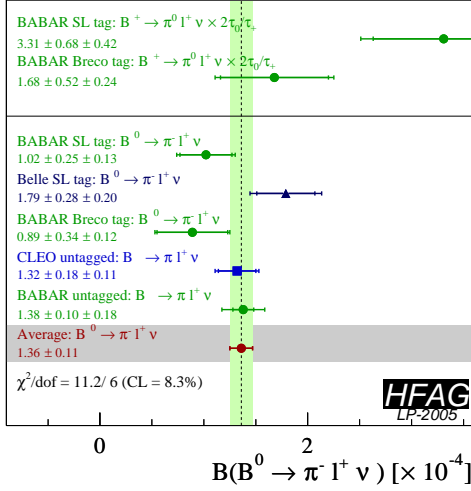


Figure 8. Summary and average of exclusive  $B \rightarrow \pi \ell \nu$  branching fractions.

Tagging the other  $B$  in the event is another powerful method to reduce backgrounds. As in the case of inclusive decays one can reconstruct the other  $B$  in an exclusive hadronic channel<sup>38</sup> (BReco tag) which allows the reconstruction of the hadronic system on the signal side. Alternatively, one can tag the other  $B$  through semileptonic decays, and use the kinematics of 2 back-to-back semileptonic decays to reduce the background.<sup>39,40,41</sup>

A summary of exclusive  $B \rightarrow \pi \ell \nu$  branching fractions measurements is shown in Fig. 8. The extraction of  $|V_{ub}|$  from these branching fractions requires a theoretical calculation of the form factor, which depends on the  $q^2$  range used. Reducing the  $q^2$  range usually improves the error on the form factor calculation while the experimental error increases because of the loss of statistics. For  $q^2 < 15 \text{ GeV}^2$  Light Cone Sum Rules<sup>23</sup> provide the most accurate calculation, whereas lattice calculation are limited to  $q^2 > 15 \text{ GeV}^2$  due to the restriction to  $\pi$  energies smaller than the inverse lattice spacing. Using the FNAL04 lattice calculations<sup>25</sup>

for  $q^2 > 16 \text{ GeV}^2$  one obtains

$$|V_{ub}|_{\text{excl.}} = (3.75 \pm 0.27^{+0.64}_{-0.42}) \times 10^{-3}.$$

It should be noted that the inclusive and exclusive determinations of  $|V_{ub}|$  are experimentally and theoretically independent. The previously reported hints of discrepancy<sup>42</sup> between the two measurements are now reduced in size and the results are compatible. Theory errors have been progressively reduced and have broken the 10% limit for the inclusive measurement.

## 5 $B \rightarrow \tau \nu$ decay

In the SM, the purely leptonic decay  $B^+ \rightarrow \ell^+ \nu$  (charge conjugate modes are implied) proceeds via the annihilation of the  $\bar{b}$  and  $u$  quark into a virtual  $W$  boson. Its amplitude is proportional to the product of  $|V_{ub}|$  and the  $B$  meson decay constant  $f_B$ , with a predicted branching fraction given by:<sup>43</sup>

$$\mathcal{B}(B^+ \rightarrow \ell^+ \nu) = \frac{G_F^2 m_B^2}{8\pi} m_\ell^2 \left(1 - \frac{m_\ell^2}{m_B^2}\right)^2 f_B^2 |V_{ub}|^2 \tau_B,$$

where  $G_F$  is the Fermi coupling constants,  $m_\ell$  and  $m_B$  are the lepton and  $B$  meson masses, and  $\tau_B$  is the  $B^+$  meson lifetime. The dependence on the lepton mass arises from helicity conservation, which suppresses the electron and muon channels. The branching ratio in the  $\tau$  channel is predicted in the SM to be roughly  $10^{-4}$ , but physics beyond the SM, such as supersymmetry or two-Higgs-doublets models could significantly modify the process. Observation of  $B \rightarrow \tau \nu$  would allow a direct determination of  $f_B$ , which is currently estimated with a 15% theoretical uncertainty<sup>44</sup> using lattice QCD calculations. Besides, the ratio of  $\mathcal{B}(B^+ \rightarrow \tau^+ \nu)$  to  $\Delta M_{B_d}$ , the mass difference between heavy and light  $B_d$  mesons, can be used to determine the ratio of  $|V_{ub}|^2/|V_{td}|^2$ , constraining an area in the  $\bar{\rho}, \bar{\eta}$  plane with small theoretical uncertainties.<sup>45</sup> Conversely, from the

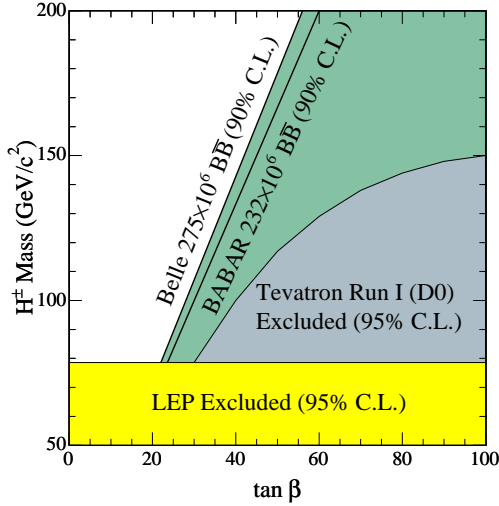


Figure 9. Exclusion are in the  $[m_H, \tan \beta]$  plane obtained from the upper limit on  $\mathcal{B}(B^+ \rightarrow \tau^+ \nu)$ .

global CKM fit one can derive<sup>21</sup> the constraint  $\mathcal{B}(B^+ \rightarrow \tau^+ \nu) = (8.1^{+1.7}_{-1.3}) \times 10^{-5}$ .

Due to the presence of at least two neutrinos in the final state, the  $B^+ \rightarrow \tau^+ \nu$  decay lacks the kinematical constraints that are usually exploited in  $B$  decay searches to reject both continuum and  $B\bar{B}$  backgrounds. The strategy adopted is to exclusively identify the other  $B$  in the event through a semileptonic or hadronic decay, and assign all the remaining tracks to the signal  $B$ . The  $\tau$  lepton is then searched in one or three prongs decays, with a maximum of one  $\pi^0$ . After applying kinematical cuts and requiring a large missing mass in the event, the most powerful variable for separating signal and background is remaining energy ( $E_{ECL}$ ) non associated with either  $B$ . Applying a cut  $E_{ECL} < 0.3 \text{ GeV}$ , Belle<sup>46</sup> finds no significant excess of events over the expected backgrounds, that ranges between 3 and 12 events depending on the  $\tau$  decay mode, and sets an upper limit  $\mathcal{B}(B^+ \rightarrow \tau^+ \nu) < 1.8 \times 10^{-4} @ 90\% \text{ C.L.}$ . BABAR finds a slightly higher upper limit.

This result can be interpreted in the context of extensions to the SM. In the two-Higgs doublet model the decay can occur via a charged Higgs particle, and the  $\mathcal{B}(B^+ \rightarrow \tau^+ \nu)$  upper limit can be translated in a constraint in the  $[m_H, \tan \beta]$  plane, as seen in Fig. 9 where  $m_H$  is the mass of the Higgs particle and  $\tan \beta$  is the ratio of the vacuum expectation values of the two Higgs doublets.<sup>47</sup>

## 6 $b \rightarrow s$ radiative decays

Radiative decays involving the  $b \rightarrow s$  flavour-changing neutral current transition occur in the SM via one-loop penguin diagrams containing an up-type quark ( $u, c, t$ ) and a  $W$  boson. Example of these decays are:  $B \rightarrow X_s \gamma, K^* \gamma, K_S^0 \pi^0 \gamma, K \pi \pi \gamma, K^{(*)} \ell^+ \ell^-, K \nu \nu, \dots$ .

New physics particles replacing the SM ones in the penguin loop, e.g. a charged Higgs boson or squarks, can affect both the total rate of these processes and the decay properties, such as photon polarization, direct CP violation, and forward-backward asymmetry in  $B \rightarrow K^{(*)} \ell^+ \ell^-$ .

### 6.1 $B \rightarrow X_s \gamma$ decays

Within the SM, the inclusive  $B \rightarrow X_s \gamma$  rate is predicted by next-to-leading order (NLO) calculations<sup>48</sup> to be  $\mathcal{B}(B \rightarrow X_s \gamma) = (3.57 \pm 0.30) \times 10^{-4}$  for  $E_\gamma > 1.6 \text{ GeV}$ . The photon energy spectrum provides access to the distribution function of the  $b$  quark inside the  $B$  meson,<sup>49</sup> whose knowledge is crucial for the extraction of  $|V_{ub}|$  from inclusive semileptonic  $B \rightarrow X_u \ell \nu$  decays, as discussed in Sec. 4. The heavy quark parameters  $m_b$  and  $\mu_\pi^2$ , which describe the effective the  $b$ -quark mass and the kinetic energy inside the  $B$  meson, can be determined from the photon energy spectrum, either by fitting the spectrum directly or by fitting the spectrum moments.<sup>50,51</sup>

The branching fraction and the photon energy spectrum can be measured with two methods, originally introduced by CLEO:<sup>52</sup>



Table 2. Summary of partial branching fraction measurements for the  $B \rightarrow X_s \gamma$  process. As explained in the text, Belle uses a photon energy cut  $E_\gamma > 1.8$  GeV, while BABAR uses  $E_\gamma > 1.9$  GeV. The errors are statistical, systematical, and model dependent.

Experiment	$\mathcal{B}(B \rightarrow X_s \gamma)[10^{-4}]$
Belle, incl. <sup>53</sup>	$3.55 \pm 0.32^{+0.30+0.11}_{-0.31-0.07}$
BABAR, incl. <sup>54</sup>	$3.67 \pm 0.29 \pm 0.34 \pm 0.29$
BABAR, excl. <sup>55</sup>	$3.27 \pm 0.18^{+0.55+0.04}_{-0.40-0.09}$

in the fully inclusive method the photon energy spectrum is measured without reconstructing the  $X_s$  system, and backgrounds are suppressed using event shape variables and high-momentum lepton tagging of the other  $B$ ; the semi-inclusive method uses a sum of exclusive final states where possible  $X_s$  systems are combined with the photon, and kinematic constraints are used to suppress backgrounds. The semi-inclusive method suffers from uncertainties on the fragmentation of the  $X_s$  system and on the assumptions made as to the fraction of unmeasured final states. On the other hand the fully-inclusive method has much larger residual backgrounds that must be carefully subtracted using off-resonance data. Table 2 summarizes the  $\mathcal{B}(B \rightarrow X_s \gamma)$  measurements. Belle uses a photon energy cut  $E_\gamma > 1.8$  GeV, while BABAR uses  $E_\gamma > 1.9$  GeV. The results are fully consistent with the SM expectations.

### 6.2 Photon polarization

In the SM, the photon from the  $b \rightarrow s \gamma$  ( $\bar{b} \rightarrow \bar{s} \gamma$ ) decays has an almost complete left-handed (right-handed) polarization. This pattern was generally assumed to be valid up to a  $O(m_s/m_b)$  correction,<sup>56</sup> but it has been recently shown<sup>57</sup> that the corrections can be significantly larger. A different polarization pattern would be a marker of new physics, and can be explored in different

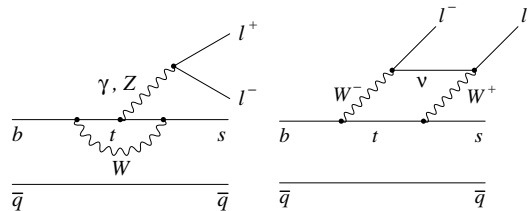


Figure 10. Feynmann diagrams describing the  $B \rightarrow s \ell^+ \ell^-$  decay.

ways. In one method<sup>56</sup> photon helicity is probed in mixing-induced CP asymmetries, exploiting the fact that left-handed and right-handed photons cannot interfere, thus suppressing time-dependent CP asymmetries in decays such as  $B \rightarrow K^{*0} \gamma$ . In another method<sup>58</sup> one uses the kaon resonances decays  $B \rightarrow K_{\text{res}} \gamma \rightarrow K \pi \pi \gamma$  to measure the up-down asymmetry of the photon direction relative to the  $K \pi \pi$  decay plane. Experimentally, many  $B \rightarrow K \pi \pi \gamma$  decay channels have been observed,<sup>59,60</sup> with branching fractions varying in the range  $(1.8 - 4.3) \times 10^{-5}$ , although with a statistics still insufficient for the helicity analysis. Both decays  $B \rightarrow K^{*0} \gamma$  and  $B \rightarrow K_S \pi^0 \gamma$  have been observed and their time-dependent CP asymmetry measured.<sup>61,62</sup> With the present statistics all the results are consistent with zero.

### 6.3 $B \rightarrow K^{(*)} \ell^+ \ell^-$ decays

As shown in Fig. 10,  $b \rightarrow s \ell^+ \ell^-$  decays proceed in the SM both via a radiative penguin diagram with a photon or a  $Z$ , and via a  $W$ -mediated box diagram. The magnitude of the photon penguin amplitude is known from the  $b \rightarrow s \gamma$  rate measurement, while the  $Z$  penguin and  $W$  box amplitudes provide new information on FCNC processes. The predicted total branching fraction is<sup>63</sup>  $\mathcal{B}(b \rightarrow s \ell^+ \ell^-) = (4.2 \pm 0.7) \times 10^{-6}$ , in agreement with measurements.<sup>64,65</sup>

The  $B \rightarrow K^{(*)} \ell^+ \ell^-$  exclusive decays are

predicted to have branching fractions of  $0.4 \times 10^{-6}$  for  $B \rightarrow K\ell^+\ell^-$  and about  $1.2 \times 10^{-6}$  for  $B \rightarrow K^*\ell^+\ell^-$ , with a theoretical uncertainty of about 30% mainly due the lack of precision in predicting how often the  $s$  quark will result in a single  $K^{(*)}$  meson in the final state. Since the electroweak couplings to electron and muon are identical, the ratio  $R_K = \mathcal{B}(B \rightarrow K\mu^+\mu^-)/\mathcal{B}(B \rightarrow Ke^+e^-)$  is expected to be unity, while in  $B \rightarrow K^*\ell^+\ell^-$  decays a phase space contribution from a pole in the photon penguin amplitude at  $q^2 = m_{\ell^+\ell^-}^2 \simeq 0$  enhances the lighter lepton pair, with a prediction of  $R_{K^*} = \mathcal{B}(B \rightarrow K^*\mu^+\mu^-)/\mathcal{B}(B \rightarrow K^*e^+e^-) = 0.752$ . Neglecting the pole region ( $q^2 < 0.1 \text{ GeV}^2$ ) for  $B \rightarrow K^*e^+e^-$ , both ratios  $R_K$  and  $R_{K^*}$  are predicted to be very close to unity. However, an enhancement of order 10% is expected in the presence of a supersymmetric neutral Higgs boson with large  $\tan\beta$ .<sup>66</sup> New physics at the electroweak scale could also enhance direct CP asymmetries, defined as  $A_{CP} = \frac{\Gamma(\bar{B} \rightarrow K^{(*)}\ell^+\ell^-) - \Gamma(B \rightarrow K^{(*)}\ell^+\ell^-)}{\Gamma(\bar{B} \rightarrow K^{(*)}\ell^+\ell^-) + \Gamma(B \rightarrow K^{(*)}\ell^+\ell^-)}$ , to values of order one,<sup>67</sup> while the SM expectations<sup>68</sup> are much less than 1%. Finally, the  $q^2$ -dependance of the lepton forward backward asymmetry is sensitive to some new physics effects, such as a change of sign<sup>69</sup> of the Wilson coefficient  $C_7$  of the Operator Product Expansion, that would not show up in other channels.

Experimentally, the  $B \rightarrow K^{(*)}\ell^+\ell^-$  decays are identified through kinematical constraints following a positive  $K$  identification. Care must be taken to reject dilepton pairs with a mass consistent with the  $J/\psi$  and the  $\psi(2S)$ , which are produced abundantly in  $B$  decays. Both processes are well established,<sup>70,71</sup> and the branching fractions are compared to theoretical calculations in Fig. 11 CP asymmetries measurements are consistent with zero with an error of 0.25. Belle also reports the first measurement of the lepton forward-backward asymmetry<sup>71</sup>

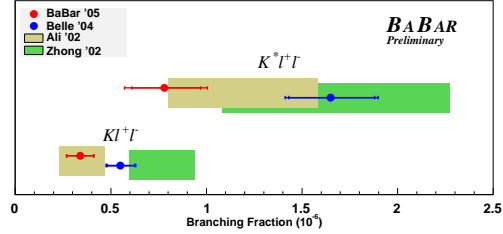


Figure 11. Experimental measurements (points) and theoretical predictions for  $B \rightarrow K^{(*)}\ell^+\ell^-$  branching fractions. Red (upper) points are the BABAR<sup>70</sup> result, while blue (lower) points are the Belle<sup>71</sup> result. The width of the boxes indicates the estimated precision of the predictions.<sup>63,73</sup>

and of the ratio of Wilson coefficients<sup>72</sup>, although the statistical power is not yet sufficient to identify new physics effects.

#### 6.4 Other radiative decays

Several other exclusive  $B$  radiative decay modes have been looked at, searching for deviation from SM expectations. No signal has been found yet, but some of the limits (given below at 90% C.L.) are getting close to the SM values.  $B \rightarrow D^{*0}\gamma$  proceeds via a  $W$ -exchange diagram and the branching fraction is expected to be around  $10^{-6}$  in the SM. The measured limit<sup>74</sup> is  $\mathcal{B}(\bar{B}^0 \rightarrow D^{*0}\gamma) < 2.5 \times 10^{-5}$ . For  $B \rightarrow \phi\gamma$ , which proceeds through a penguin annihilation diagram<sup>75</sup> the SM expectations are around  $10^{-12}$ , while the experimental limit is  $\mathcal{B}(B^0 \rightarrow \phi\gamma) < 8.5 \times 10^{-7}$ . The double radiative decay  $B \rightarrow \gamma\gamma$  has a clean experimental signature and is expected to be around  $3 \times 10^{-8}$  in the SM. The measurements<sup>76</sup> limit its rate at  $\mathcal{B}(B^0 \rightarrow \gamma\gamma) < 5.4 \times 10^{-7}$

### 7 Observation of $b \rightarrow d$ radiative decays

The  $b \rightarrow d\gamma$  process is suppressed with respect to  $b \rightarrow s\gamma$  by a factor  $|V_{td}/V_{ts}|^2 \simeq 0.04$ . Due to the large background from continuum

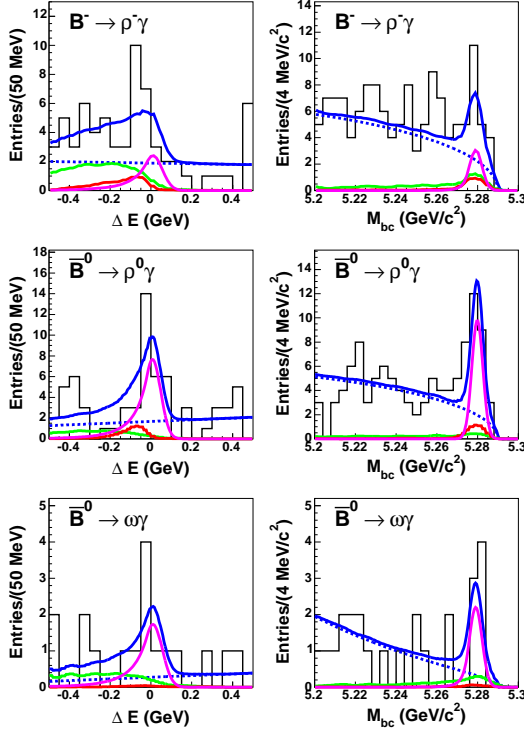


Figure 12. Projection of the fit results to  $M_{bc}$  and  $\Delta E$  for the individual  $b \rightarrow d\gamma$  modes. Lines represent the signal (magenta), continuum (blue-dashed),  $B \rightarrow K^*\gamma$  (red), other  $B$  decay background components (green), and the total fit result (blue-solid).

events, only exclusive modes such as  $B^- \rightarrow \rho^- \gamma$ ,  $\bar{B}^0 \rightarrow \rho^0 \gamma$ ,  $\bar{B}^0 \rightarrow \omega \gamma$  (charge conjugate modes are implied), have been searched so far. Measurement of these exclusive branching fractions, which are predicted to be in the range  $(0.9 - 2.7) \times 10^{-6}$  in the SM,<sup>77</sup> gives a precise determination of  $|V_{td}/V_{ts}|$  and provides sensitivity to physics beyond the SM. Belle reports the first observation<sup>78</sup> of these decays, reconstructing the  $\rho$  and  $\omega$  with final states with at most one  $\pi^0$ . Background rejection is obtained through the use of event shape variables, vertex separation, and by tagging the other  $B$  in the event. All the variables are used in an unbinned maximum likelihood fit where the  $B \rightarrow (\rho, \omega)\gamma$  and  $B \rightarrow K^*\gamma$  yields are simultaneously determined.

Figure 12 shows the projection of the

likelihood fit onto the  $M_{bc}$  and  $\Delta E$  axes for the individual modes. A clear peak is always visible. The individual branching ratios are determined as follows:

$$\begin{aligned} \mathcal{B}(B^- \rightarrow \rho^- \gamma) &= (0.55^{+0.43+0.12}_{-0.37-0.11}) \times 10^{-6}, \\ \mathcal{B}(\bar{B}^0 \rightarrow \rho^0 \gamma) &= (1.17^{+0.35+0.09}_{-0.31-0.08}) \times 10^{-6}, \\ \mathcal{B}(\bar{B}^0 \rightarrow \omega \gamma) &= (0.58^{+0.35+0.07}_{-0.27-0.11}) \times 10^{-6}, \end{aligned}$$

where the first error is statistical and the second error is systematical. The significance figures of the three measurements are  $1.5\sigma$ ,  $5.1\sigma$ , and  $2.6\sigma$ , respectively. A simultaneous fit is also performed using the isospin relation:

$$\begin{aligned} \mathcal{B}(B \rightarrow \rho/\omega \gamma) &\equiv \mathcal{B}(B^- \rightarrow \rho^- \gamma) = \\ 2 \frac{\tau_{B^+}}{\tau_{B^0}} \mathcal{B}(\bar{B}^0 \rightarrow \rho^0 \gamma) &= 2 \frac{\tau_{B^+}}{\tau_{B^0}} \mathcal{B}(\bar{B}^0 \rightarrow \omega \gamma) \end{aligned}$$

where  $\tau_{B^+}/\tau_{B^0} = 1.076 \pm 0.008$  is the ratio of charged  $B$  lifetime to the neutral  $B$  lifetime, yielding

$$\mathcal{B}(B \rightarrow \rho/\omega \gamma) = 1.34^{+0.345+0.14}_{-0.31-0.10} \quad (5.5\sigma)$$

It should be noted that the individual fit results (especially  $\mathcal{B}(B \rightarrow \rho^0 \gamma)$ ) are in marginal agreement with the isospin relation above or with the previous limits. More statistics will hopefully clarify the issue. The simultaneous determination of  $\mathcal{B}(B \rightarrow K^* \gamma)$  allows the determination of  $|V_{td}/V_{ts}|$ :<sup>79</sup>

$$|V_{td}/V_{ts}| = 0.200^{+0.026+0.038}_{-0.025-0.029},$$

where the errors are respectively from experiment and theory. This value is in agreement with global fit to the unitarity triangle,<sup>21</sup> but the  $b \rightarrow d\gamma$  observation provides an independent constraint on the unitarity triangle which will become more and more effective as statistics increase.

## 8 Summary and conclusions

The accuracy of the analyses performed by the *BABAR* and *Belle* experiments has been steadily improving, and the precision measurement of CKM parameters is now a reality.



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